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FURTHER STABILITY ANALYSIS OF NEUTRAL-TYPE COHEN-GROSSBERG NEURAL NETWORKS WITH MULTIPLE DELAYS

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Abstract. The key contribution of this paper is to study the stability analysis of neutral-type Cohen-Grossberg neural networks possessing multiple time delays in the states of the neurons and multiple neutral delays in time derivative of states of the neurons. By making the use of a proper Lyapunov functional, we propose a novel sufficient time-independent stability criterion for this model of neutral-type neural networks. The proposed stability criterion in this paper can be absolutely expressed in terms of the parameters of the neural network model considered as this newly proposed criterion only relies on the relationships established among the network parameters. A numerical example is also given to indicate the advantages of the obtained stability criterion over the previously published stability results for the same class of Cohen-Grossberg neural networks. Since obtaining stability conditions for neutral-type Cohen-Grossberg neural networks with multiple delays is a difficult task to achieve, there are only few papers in the literature dealing with this problem. Therefore, the results given in the current paper makes an important contribution to the stability problem for this class of neutral-type neural networks.

1. Introduction.

In recent decades, various classes of neural networks have been employed to solve many different engineering problems arising in the real world applications such as moving image processing, control and optimization applications, parallelly computing systems, associative memory design, (The readers can refer to the references [1]-[7] for real world application of neural systems). In the design of neural networks for solving practical engineering problems, it is crucial to address the stability and equilibrium characterison of these designed neural systems. In particular, in the electronic implementation of neural systems using VLSI technology, due to the finite switching speed of amplifiers and the communication times among the neurons bring about some unavoidable time delays, which can change the aimed dynamical properties of neural systems. Beacuse of such dynamical problems caused by time delays, it is a critical task to investigate the stability criteria for neural networks whose dynamical model involve delay parameters. In the recent literature, many various stability results have been proposed, which establish the global asymptotic stability of different classes of neural systems in the presence of

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time delays [8]-[25]. On the other hand, it is known that including the delay parameters in the time derivative of states of the neurons enables us to determine a complete characterization of dynamical properties of delayed neural systems. Such a modification in neural networks leads us to establish the delayed neutral-type neural network models. Such neutral-type systems have useful applications in population ecology, distributed networks with loss less transmission lines, propagation and diffusion models [26]-[28].

This paper will consider the neutral-type Cohen-Grossberg neural networks possessing multiple time delays in the states of the neurons and multiple neutral delays in time derivatives of the states of the neurons described by the following sets of differential equations :

$$\begin{aligned}\dot{x}_i(t) =& d_i(x_i(t))\Big(- c_i(x_i(t)) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_{ij})) + u_i\Big) \\ &+ \sum_{j=1}^{n} e_{ij}\dot{x}_j(t-\zeta_{ij}), \quad i = 1, \cdots, n.\end{aligned} \tag{1}$$

in which $x_i(t)$ denotes the state of the ith neuron, $c_i(x_i)$ are some behaved functions and $d_i(x_i(t))$ represent the amplification functions. The constant parameters a_{ij} and b_{ij} represent the values of the interconnections among the neurons. τ_{ij} $(1 \leq i, j \leq n)$ represent the time delay parameters and ζ_{ij} $(1 \leq i, j \leq n)$ represent the neutral delay parameters. The parameters e_{ij} denote the coefficients of the time derivative of the states involving delays. The nonlinear activation functions are denoted by $f_j(\cdot)$ and u_i are constant inputs. In (1), if assume that $\tau = max\{\tau_{ij}\}$, $\zeta = max\{\zeta_{ij}\}$, $1 \leq i, j \leq n$, and $\xi = max\{\tau, \zeta\}$. In this case, neural system (1) has the initial conditions given by : $x_i(t) = \varphi_i(t)$ and $\dot{x}_i(t) = \vartheta_i(t) \in C([-\xi, 0], R)$ with $C([-\xi, 0], R)$ being the set of all continuous functions from $[-\xi, 0]$ to R.

Before proceeding with the stability analysis of neutral system (1), we need to give the properties of $d_i(x_i(t))$, $c_i(x_i(t))$ and $f_i(x_i(t))$. These functions are assumed to possess following main conditions :

A_1 : For the functions $d_i(x_i(t))$, there exist positive real numbers μ_i and ρ_i such that the following conditions hold :

$$0 < \mu_i \leq d_i(x_i(t)) \leq \rho_i, \ i = 1, 2, ..., n, \ \forall x_i(t) \in R.$$

A_2 : For the functions $c_i(x_i(t))$, there exist positive real numbers γ_i and ψ_i such that the following conditions hold :

$$\begin{aligned}&0 < \gamma_i \leq \frac{c_i(x_i(t)) - c_i(y_i(t))}{x_i(t) - y_i(t)} = \frac{|c_i(x_i(t)) - c_i(y_i(t))|}{|x_i(t) - y_i(t)|} \leq \psi_i, \\ &\forall x_i(t), y_i(t) \in R, x_i(t) \neq y_i(t), \ i = 1, 2, ..., n.\end{aligned}$$

A_3 : For the functions $f_i(x_i(t))$ there exist positive real numbers ℓ_i such that the following conditions hold :

$$|f_i(x_i(t)) - f_i(y_i(t))| \leq \ell_i |x_i(t) - y_i(t)|, \quad \forall x_i(t), y_i(t) \in R, x_i(t) \neq y_i(t), \ i = 1, 2, ..., n.$$

Since, neutral-type neural system (1) cannot be stated in the matrix-vector form due to involving multiple delays, studying the stabilty of system (1) has been a very difficult problem to overcome. Therefore, in the past literature, many researchers have focused on the stability analysis of a special model of system (1), which is described by the following sets of equations :

$$\begin{aligned}\dot{x}_i(t) =& d_i(x_i(t))\Big(- c_i(x_i(t)) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j)) + u_i\Big) \\ &+ \sum_{j=1}^{n} e_{ij}\dot{x}_j(t-\zeta_j), \quad i = 1, \cdots, n.\end{aligned} \tag{2}$$

Note that neutral system (2) is a specialized model of neutral system (1) with the assumptions that $\tau_{ij} = \tau_j$ and $\zeta_{ij} = \zeta_j$, $\forall i, j$. Note that system (2) can be mathematically expressed in the vector-matrix form. This makes it possible to develope and employ some suitable Lyapunov functionals to study the stability problem for system (2). There exist many results ensuring the stability of system (2) in the past literature [29]-[42] where various forms of neutral system (2) have been considered. In these papers, the proposed results have been derived by employing various and modified Lyapunov functionals and these stability results have been expressed in the various forms of linear-matrix inequalities (LMIs). On the other hand, some recent papers have presented some new algebraic stability criteria which can be considered as alternative results to those that are in the LMI forms [43]-[47]. To the best of the knowledge of the author of this paper, only a recent paper has presented some results on the stability of neutral system (1) [48]. In our current paper, by developing a novel Lyapunov functional, we will study neutral system (1) and derive some new and alternative global asymptotic stability conditions for this system.

2. **Stability analysis.** This section will deal with determining a new criterion that guarantees the global stability of the equilibrium point of delayed Cohen-Groosberg neural system of neutral-type given by (1). In order to achieve this task, the equilibrium points $x^* = (x_1^*, x_2^*, ..., x_n^*)^T$ of delayed neutral system (1) is to be shifted to the origin. This is usually done by using the transformation $z_i(t) = x_i(t) - x_i^*$, which can deduce the neutral system of the form

$$\begin{aligned}\dot{z}_i(t) =& \alpha_i(z_i(t))\Big(- \beta_i(z_i(t)) + \sum_{j=1}^{n} a_{ij} g_j(z_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big) \\ &+ \sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij}), \quad i = 1, \cdots, n.\end{aligned} \tag{3}$$

where $\alpha_i(z_i(t)) = d_i(z_i(t) + x_i^*)$, $\beta_i(z_i(t)) = c_i(z_i(t) + x_i^*) - c_i(x_i^*)$, and $g_i(z_i(t)) = f_i(z_i(t) + x_i^*) - f_i(x_i^*), \forall i$ It should be pointed out that neural-type neural system defined (3) possesses the properties of the assumptions A_1, A_2 and A_3. We can restate these assumptions for system (3) as follows :

$\tilde{A}_1$: $\mu_i \leq \alpha_i(z_i(t)) \leq \rho_i$, $\forall i$,

$\tilde{A}_2$: $\gamma_i z_i^2(t) \leq z_i(t)\beta_i(z_i(t)) \leq \psi_i z_i^2(t)$, $\forall i$,

$\tilde{A}_3$: $|g_i(z_i(t))| \leq \ell_i |z_i(t)|$, $\forall i$.

We are now in the position to derive the main condition for the global asymptotic stability of system (1) which is given in the following theorem :

Theorem 2.1. *Let the neutral-type neural system described by (3) satisfy the assumptions $\tilde{A}_1 - \tilde{A}_3$. Then, the origin of neural system (3) is globally asymptotically* *stable if there exist positive constants δ and κ such that the following conditions hold:*

$$\nu_i = \gamma_i^2 - (2+\kappa)n\sum_{j=1}^{n} a_{ji}^2\ell_i^2 - (2+\delta)n\sum_{j=1}^{n} b_{ji}^2\ell_i^2 > 0, \ i = 1,\cdots,n$$

$$\upsilon_{ij} = \frac{1}{n}\frac{1}{\rho_j^2} - (1+\frac{1}{\kappa}+\frac{1}{\delta})n\frac{1}{\mu_i^2}e_{ij}^2 > 0, \ i,j = 1,\cdots,n.$$

Proof : This theorem will be proved by utilizing the following positive definite Lyapunov functional :

$$\begin{aligned} V(t) \quad = \quad & 2\sum_{i=1}^{n}\int_{0}^{z_i(t)}\frac{\beta_i(s)}{\alpha_i(s)}ds + \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\zeta_{ij}}^{t}\frac{1}{\alpha_j^2(z_j(s))}\dot{z}_j^2(s)ds \\ & + n(2+\delta)\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}}^{t} b_{ij}^2 g_j^2(z_j(s))ds + \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}}^{t} z_j^2(s)ds \end{aligned}$$

where ε is a positive constant whose numerical value will be determined later. The time derivative of $V(t)$ along the trajectories of the neutral-type neural system described by (3) is derived to be in the following form :

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^{n} 2\frac{\beta_i(z_i(t))}{\alpha_i(z_i(t))}\dot{z}_i(t) + \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\alpha_j^2(z_j(t))}\dot{z}_j^2(t) \\ & - \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\alpha_j^2(z_j(t-\zeta_{ij}))}\dot{z}_j^2(t-\zeta_{ij}) \\ & + n(2+\delta)\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^2 g_j^2(z_j(t)) \\ & - n(2+\delta)\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^2 g_j^2(z_j(t-\tau_{ij})) \\ & + \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2(t) - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2(t-\tau_{ij}) \end{aligned} \tag{4}$$

We can write the following equality :

$$\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\alpha_j^2(z_j(t))}\dot{z}_j^2(t) = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\alpha_i^2(z_i(t))}\dot{z}_i^2(t) = \sum_{i=1}^{n}\frac{1}{\alpha_i^2(z_i(t))}\dot{z}_i^2(t) \tag{5}$$

Using (5) in (4) results in

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^{n} 2\frac{\beta_i(z_i(t))}{\alpha_i(z_i(t))}\dot{z}_i(t) + \sum_{i=1}^{n}\frac{1}{\alpha_i^2(z_i(t))}\dot{z}_i^2(t) \\ & - \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\alpha_j^2(z_j(t-\zeta_{ij}))}\dot{z}_j^2(t-\zeta_{ij}) \\ & + n(2+\delta)\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^2 g_j^2(z_j(t)) \end{aligned}$$

$$\begin{aligned}
&- n(2+\delta)\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^2 g_j^2(z_j(t-\tau_{ij}))\\
&+ \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2(t) - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2(t-\tau_{ij})\\
=&\sum_{i=1}^{n}\left(2\beta_i(z_i(t)) + \frac{\dot z_i(t)}{\alpha_i(z_i(t))}\right)\frac{\dot z_i(t)}{\alpha_i(z_i(t))}\\
&- \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\alpha_j^2(z_j(t-\zeta_{ij}))}\dot z_j^2(t-\zeta_{ij}) \qquad (6)\\
&+ n(2+\delta)\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^2 g_j^2(z_j(t))\\
&- n(2+\delta)\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^2 g_j^2(z_j(t-\tau_{ij}))\\
&+ \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2(t) - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2(t-\tau_{ij})
\end{aligned}$$

From (3), we can write the equality :

$$\begin{aligned}
\frac{\dot z_i(t)}{\alpha_i(z_i(t))} =& - \beta_i(z_i(t)) + \sum_{j=1}^{n} a_{ij}g_j(z_j(t)) + \sum_{j=1}^{n} b_{ij}g_j(z_j(t-\tau_{ij}))\\
&+ \frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot z_j(t-\zeta_{ij}), \quad i=1,\cdots,n.
\end{aligned} \tag{7}$$

Adding the term $2\beta_i(z_i(t))$ to the both sides of (7) leads to

$$\begin{aligned}
2\beta_i(z_i(t)) + \frac{\dot z_i(t)}{\alpha_i(z_i(t))} =& \beta_i(z_i(t)) + \sum_{j=1}^{n} a_{ij}g_j(z_j(t)) + \sum_{j=1}^{n} b_{ij}g_j(z_j(t-\tau_{ij}))\\
&+ \frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot z_j(t-\zeta_{ij}), \quad i=1,\cdots,n.
\end{aligned} \tag{8}$$

Multiplying (7) by (8) results in

$$\begin{aligned}
&(2\beta_i(z_i(t)) + \frac{\dot z_i(t)}{\alpha_i(z_i(t))})\frac{\dot z_i(t)}{\alpha_i(z_i(t))}\\
&= \Bigg(- \beta_i(z_i(t)) + \sum_{j=1}^{n} a_{ij}g_j(z_j(t)) + \sum_{j=1}^{n} b_{ij}g_j(z_j(t-\tau_{ij}))\\
&+ \frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot z_j(t-\zeta_{ij})\Bigg) \times \Bigg(\beta_i(z_i(t)) + \sum_{j=1}^{n} a_{ij}g_j(z_j(t))\\
&+ \sum_{j=1}^{n} b_{ij}g_j(z_j(t-\tau_{ij})) + \frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot z_j(t-\zeta_{ij})\Bigg)
\end{aligned}$$

$$
\begin{aligned}
&= - \beta_i^2(z_i(t)) + \Big(\sum_{j=1}^{n} a_{ij} g_j(z_j(t))\Big)^2 + \Big(\sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big)^2 \\
&\quad + \Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big)^2 \\
&\quad + 2\Big(\sum_{j=1}^{n} a_{ij} g_j(z_j(t))\Big)\Big(\sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big) \\
&\quad + 2\Big(\sum_{j=1}^{n} a_{ij} g_j(z_j(t))\Big)\Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big) \\
&\quad + 2\Big(\sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big)\Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big)
\end{aligned} \tag{9}
$$

We now note the following inequalities :

$$
\begin{aligned}
&2\Big(\sum_{j=1}^{n} a_{ij} g_j(z_j(t))\Big)\Big(\sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big) \\
&\leq \Big(\sum_{j=1}^{n} a_{ij} g_j(z_j(t))\Big)^2 + \Big(\sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big)^2
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
&2\Big(\sum_{j=1}^{n} a_{ij} g_j(z_j(t))\Big)\Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big) \\
&\leq \kappa\Big(\sum_{j=1}^{n} a_{ij} g_j(z_j(t))\Big)^2 + \frac{1}{\kappa}\Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big)^2
\end{aligned} \tag{11}
$$

and

$$
\begin{aligned}
&2\Big(\sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big)\Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big) \\
&\leq \delta\Big(\sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big)^2 + \frac{1}{\delta}\Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big)^2
\end{aligned} \tag{12}
$$

where κ and δ are some positive constants. Using (10)-(12) in (9) yields :

$$
\begin{aligned}
&(2\beta_i(z_i(t)) + \frac{\dot{z}_i(t)}{\alpha_i(z_i(t))})\frac{\dot{z}_i(t)}{\alpha_i(z_i(t))} \\
&\leq -\beta_i^2(z_i(t)) + 2\Big(\sum_{j=1}^{n} a_{ij} g_j(z_j(t))\Big)^2 + 2\Big(\sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big)^2 \\
&+ \kappa\Big(\sum_{j=1}^{n} a_{ij} g_j(z_j(t))\Big)^2 + \delta\Big(\sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big)^2 \\
&+ \frac{1}{\kappa}\Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big)^2 + \frac{1}{\delta}\Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big)^2 \\
&+ \Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big)^2
\end{aligned} \tag{13}
$$

$$= -\beta_i^2(z_i(t)) + (2+\kappa)\Big(\sum_{j=1}^{n} a_{ij} g_j(z_j(t))\Big)^2$$
$$+ (2+\delta)\Big(\sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big)^2$$
$$+ (1+\frac{1}{\kappa}+\frac{1}{\delta})\Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big)^2$$

Note the inequalities :

$$\Big(\sum_{j=1}^{n} a_{ij} g_j(z_j(t))\Big)^2 \le n\sum_{j=1}^{n} a_{ij}^2 g_j^2(z_j(t)) \tag{14}$$

$$\Big(\sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_{ij}))\Big)^2 \le n\sum_{j=1}^{n} b_{ij}^2 g_j^2(z_j(t-\tau_{ij})) \tag{15}$$

$$\Big(\frac{1}{\alpha_i(z_i(t))}\sum_{j=1}^{n} e_{ij}\dot{z}_j(t-\zeta_{ij})\Big)^2 \quad \le \quad n\sum_{j=1}^{n}\frac{1}{\alpha_i^2(z_i(t))} e_{ij}^2 \dot{z}_j^2(t-\zeta_{ij}) \tag{16}$$

Using (14)-(16) in (13) leads to :

$$\begin{aligned}(2\beta_i(z_i(t)) + \frac{\dot{z}_i(t)}{\alpha_i(z_i(t))})\frac{\dot{z}_i(t)}{\alpha_i(z_i(t))} = &- \beta_i^2(z_i(t)) + (2+\kappa)n\sum_{j=1}^{n} a_{ij}^2 g_j^2(z_j(t))\\ &+ (2+\delta)n\sum_{j=1}^{n} b_{ij}^2 g_j^2(z_j(t-\tau_{ij}))\\ &+ (1+\frac{1}{\kappa}+\frac{1}{\delta})\sum_{j=1}^{n}\frac{n}{\alpha_i^2(z_i(t))} e_{ij}^2 \dot{z}_j^2(t-\zeta_{ij})\end{aligned} \tag{17}$$

Thus, from (17), we can write

$$\begin{aligned}&\sum_{i=1}^{n}(2\beta_i(z_i(t)) + \frac{\dot{z}_i(t)}{\alpha_i(z_i(t))})\frac{\dot{z}_i(t)}{\alpha_i(z_i(t))}\\ &\le -\sum_{i=1}^{n}\beta_i^2(z_i(t)) + (2+\kappa)n\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^2 g_j^2(z_j(t))\\ &+ (2+\delta)n\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^2 g_j^2(z_j(t-\tau_{ij}))\\ &+ (1+\frac{1}{\kappa}+\frac{1}{\delta})n\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\alpha_i^2(z_i(t))} e_{ij}^2 \dot{z}_j^2(t-\zeta_{ij})\end{aligned} \tag{18}$$

Using (18) in (6) will give the following :

$$\dot{V}(t) \le -\sum_{i=1}^{n}\beta_i^2(z_i(t)) + (2+\kappa)n\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^2 g_j^2(z_j(t))$$
$$+ (2+\delta)n\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}^2 g_j^2(z_j(t))$$

$$
\begin{aligned}
&+ (1+\frac{1}{\kappa}+\frac{1}{\delta})n\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\alpha_i^2(z_i(t))}e_{ij}^2\dot{z}_j^2(t-\zeta_{ij})\\
&-\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\alpha_j^2(z_j(t-\zeta_{ij}))}\dot{z}_j^2(t-\zeta_{ij})\\
&+\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t)-\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij})\\
&=-\sum_{i=1}^{n}\beta_i^2(z_i(t))+(2+\kappa)n\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ji}^2g_i^2(z_i(t))\\
&+(2+\delta)n\sum_{i=1}^{n}\sum_{j=1}^{n}b_{ji}^2g_i^2(z_i(t))\\
&+(1+\frac{1}{\kappa}+\frac{1}{\delta})n\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\alpha_i^2(z_i(t))}e_{ij}^2\dot{z}_j^2(t-\zeta_{ij})\\
&-\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\alpha_j^2(z_j(t-\zeta_{ij}))}\dot{z}_j^2(t-\zeta_{ij})\\
&+\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t)-\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij})
\end{aligned}
\tag{19}
$$

Under the assumptions $\tilde{A}_1-\tilde{A}_3$, we have $\alpha_j^2(z_j(t-\tau_{ij}))\le\rho_j^2$, $\alpha_i^2(z_i(t))\ge\mu_i^2$, $\beta_i^2(z_i(t))\ge\gamma_i^2z_i^2(t)$ and $g_i^2(z_i(t))\le\ell_i^2z_i^2(t)$, $i,j=1,2,\cdots,n$. Thus, (19) can be written as follows :

$$
\begin{aligned}
\dot{V}(t)\le&-\sum_{i=1}^{n}\gamma_i^2z_i^2(t)+(2+\kappa)n\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}^2\ell_j^2z_j^2(t)\\
&+(2+\delta)n\sum_{i=1}^{n}\sum_{j=1}^{n}b_{ij}^2\ell_j^2z_j^2(t)\\
&+(1+\frac{1}{\kappa}+\frac{1}{\delta})n\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\mu_i^2}e_{ij}^2\dot{z}_j^2(t-\zeta_{ij})\\
&-\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\rho_j^2}\dot{z}_j^2(t-\zeta_{ij})+\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t)-\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij})\\
=&-\sum_{i=1}^{n}\gamma_i^2z_i^2(t)+(2+\kappa)n\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ji}^2\ell_i^2z_i^2(t)\\
&+(2+\delta)n\sum_{i=1}^{n}\sum_{j=1}^{n}b_{ji}^2\ell_i^2z_i^2(t)\\
&+(1+\frac{1}{\kappa}+\frac{1}{\delta})n\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\mu_i^2}e_{ij}^2\dot{z}_j^2(t-\zeta_{ij})-\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\rho_j^2}\dot{z}_j^2(t-\zeta_{ij})\\
&+\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t)-\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij})
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
&= -\sum_{i=1}^{n}(\gamma_i^2 - (2+\kappa)n\sum_{j=1}^{n}a_{ji}^2\ell_i^2 - (2+\delta)n\sum_{j=1}^{n}b_{ji}^2\ell_i^2)z_i^2(t) \\
&- \sum_{i=1}^{n}\sum_{j=1}^{n}(\frac{1}{n}\frac{1}{\rho_j^2} - (1+\frac{1}{\kappa}+\frac{1}{\delta})n\frac{1}{\mu_i^2}e_{ij}^2)\dot{z}_j^2(t-\zeta_{ij}) \\
&+ \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t) - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij}) \\
&= -\sum_{i=1}^{n}\nu_i z_i^2(t) - \sum_{i=1}^{n}\sum_{j=1}^{n}\upsilon_{ij}\dot{z}_j^2(t-\zeta_{ij}) \\
&+ \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t) - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij})
\end{aligned}
$$

Since $\upsilon_{ij} > 0$, $\forall i, j$ and $\varepsilon > 0$, (20) takes the form :

$$
\begin{aligned}
\dot{V}(t) &\le -\sum_{i=1}^{n}\nu_i z_i^2(t) + \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t) \\
&= -\sum_{i=1}^{n}\nu_i z_i^2(t) + n\varepsilon\sum_{i=1}^{n}z_i^2(t) \\
&\le -\nu_m||z(t)||_2^2 + n\varepsilon||z(t)||_2^2 \\
&= -(\nu_m - n\varepsilon)||z(t)||_2^2
\end{aligned}
\tag{21}
$$

where $\nu_m = min\{\nu_i\}$ and $z(t) = (z_1(t), z_2(t), \cdots, z_n(t))^T$. In (21), the choice $\varepsilon < \frac{\nu_m}{n}$ will guarantee that $\dot{V}(t) < 0$ for all $z(t) \neq 0$. Now, consider the case where $z(t) = 0$. (Note that $z_i(t) = 0$ implies that $g_i(z_i(t)) = 0$). In this case, from (20), we obtain

$$
\begin{aligned}
\dot{V}(t) &\le -\sum_{i=1}^{n}\sum_{j=1}^{n}\upsilon_{ij}\dot{z}_j^2(t-\zeta_{ij}) - \varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij}) \\
&\le -\varepsilon\sum_{i=1}^{n}\sum_{j=1}^{n}z_j^2(t-\tau_{ij})
\end{aligned}
\tag{22}
$$

It follows from (22) that if $z_j(t-\tau_{ij}) \neq 0$ for any randomly selected pairs of i and j, then, $\dot{V}(t)$ will be strictly negative definite. Now, consider the case where $z(t) = 0$ and $z_j(t-\tau_{ij}) = 0$, $i, j = 1, 2, ..., n$. (Note that $z_j(t-\tau_{ij}) = 0$ implies that $g_j(z_j(t-\tau_{ij})) = 0$). In this case, from (20), we obtain

$$
\dot{V}(t) \le -\sum_{i=1}^{n}\sum_{j=1}^{n}\upsilon_{ij}\dot{z}_j^2(t-\zeta_{ij}) \tag{23}
$$

Since $\upsilon_{ij} > 0$, $\forall i, j$, it follows from (23) that if $\dot{z}_j(t-\zeta_{ij}) \neq 0$ for any randomly selected pairs of i and j, then, $\dot{V}(t)$ will be strictly negative definite. Now, consider the case where $z_i(t) = 0$, $g_i(z_i(t)) = 0$, $z_j(t-\tau_{ij}) = 0$, $g_j(z_j(t-\tau_{ij})) = 0$ and $\dot{z}_j(t-\zeta_{ij}) = 0$, $i, j = 1, 2, \cdots, n$. In this case, we have

$$
\dot{V}(t) = \sum_{i=1}^{n}\frac{1}{\alpha_i^2(z_i(t))}\dot{z}_i^2(t) \tag{24}
$$

Note that if $z_i(t) = 0$, $g_i(z_i(t)) = 0$, $z_j(t-\tau_{ij}) = 0$, $g_j(z_j(t-\tau_{ij})) = 0$ and $\dot{z}_j(t-\zeta_{ij}) = 0$, $i,j = 1,2,\cdots,n$, then, from (3), we have $\dot{z}_i(t) = 0$, $\forall i$. Therefore, in this case, $\dot{V}(t) = 0$. Thus, one can directly see that the condition $\dot{V}(t) = 0$ holds iff when $z_i(t) = 0$, $g_i(z_i(t)) = 0$, $z_j(t-\tau_{ij}) = 0$, $g_j(z_j(t-\tau_{ij})) = 0$ and $\dot{z}_j(t-\zeta_{ij}) = 0$, $i,j = 1,2,\cdots,n$, and $\dot{V}(t)$ is negative definite for all the other cases. Based on the above analysis of the time derivative of the Lyapunov functional used in the stability analysis, it can be stated that the origin of neutral-type neural model (3) is asymptotically stable. It is also worth noting that the employed Lyapunov functional is radially unbounded, that is to say, $V(z(t)) \to \infty$ as $||z(t)|| \to \infty$. This property of the Lyapunov functional ensures that the origin of neutral-type neural model (3) is globally asymptotically stable. Thus, one can directly conclude that the equilibrium point of neutral-type neural model (1) is globally asymptotically stable.

3. **Comparisons and an example.**

The following theorem has been given in [48]:

Theorem 3.1. *Let the neutral-type neural system described by (3) satisfy the assumptions $\tilde{A}_1 - \tilde{A}_3$. Then, the origin of neural system (3) is globally asymptotically stable if the following conditions hold :*

$$\begin{aligned}\varepsilon_i \;=\; & 2\mu_i\gamma_i - \sum_{j=1}^{n}(\rho_i\ell_j|a_{ij}| + \rho_j\ell_i|a_{ji}|) - \sum_{j=1}^{n}(\rho_i\ell_j|b_{ij}| + \rho_j\ell_i|b_{ji}|) \\ & - \sum_{j=1}^{n}(\rho_i\psi_i|e_{ij}| + \rho_j\psi_j|e_{ji}|) - \sum_{j=1}^{n}\sum_{k=1}^{n}(\rho_i\ell_i|a_{ki}||e_{kj}| + \rho_k\ell_i|b_{ki}||e_{kj}|) \\ & - \sum_{j=1}^{n}\sum_{k=1}^{n}(\rho_j\ell_k|a_{jk}||e_{ji}| + \rho_j\ell_k|b_{jk}||e_{ji}|) > 0,\ \forall i\end{aligned}$$

and

$$\epsilon_i \;=\; 1 - \sum_{j=1}^{n}|e_{ji}| > 0,\ \forall i.$$

Then, the origin of neutral-type system (1) is globally asymptotically stable.

We now study the following example to exploit the effectiveness and advantages of the criterion proposed in Theorem 2.1.

Example. Consider the neutral-type neural network model given by (1) with the following system parameters :

$$a_{ij} = \frac{1}{16},\ b_{ij} = \frac{1}{16},\ \text{and}\ e_{ij} = e,\ i,j = 1,2,3,4.$$
$$\rho_1 = \rho_2 = \rho_3 = \rho_4 = 1, \ell_1 = \ell_2 = \ell_3 = \ell_4 = 1$$
$$\mu_1 = \mu_2 = \mu_3 = \mu_4 = 1, \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 1,1,$$
$$\psi_1 = \psi_2 = \psi_3 = \psi_4 = \psi$$

Let us first apply the results of Theorem 3.1 to this example to derive the stability conditions. The conditions of Theorem 3.1 are obtained as follows:

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 \;=\; 2,2 - \sum_{j=1}^{4}(|a_{ij}| + |a_{ji}|) - \sum_{j=1}^{4}(|b_{ij}| + |b_{ji}|)$$

$$
\begin{aligned}
& -\sum_{j=1}^{4} \psi(|e_{ij}| + |e_{ji}|) \\
& -\sum_{j=1}^{4}\sum_{k=1}^{4}(|a_{ki}||e_{kj}| + |b_{ki}||e_{kj}|) \\
& -\sum_{j=1}^{4}\sum_{k=1}^{4}(|a_{jk}||e_{ji}| + |b_{jk}||e_{ji}|) \\
= \;& 1,2 - 8\psi e - 4e > 0
\end{aligned}
$$

from which the stability condition of Theorem 3.1 is derived as follows :

$$e < \frac{1,2}{8\psi + 4}$$

According to assumption $\tilde{A}_2$, for this example, the minimum value of ψ is $\psi = 1,1$. For this value of ψ, e must satisfy the conditon $e < 0,09375$.

We now apply the results of Theorem 2.1 to this example to derive the stability conditions. For $\kappa = 6$ and $\delta = 6$, the conditions of Theorem 2.1 are obtained as follows :

$$
\begin{aligned}
\nu_i &= 1,21 - 8n\sum_{j=1}^{n} a_{ji}^2 - 8n\sum_{j=1}^{n} b_{ji}^2 \\
&= 0,21 > 0, \;\; i = 1,2,3,4 \\
\upsilon_{ij} &= \frac{1}{n} - \frac{4}{3}ne^2 > 0, \;\; i,j = 1,2,3,4.
\end{aligned}
$$

from which the stability condition of Theorem 2.1 is derived as follows :

$$e < \frac{\sqrt{3}}{8}$$

implying that $e < 0,2165$ is a sufficient condition for the stability of the system given in this example. When we compare stability conditions of this example imposed by Theorems 2.1 and 3.1, we can see that Theorem 2.1 imposes a less restrictive stability condition than Theorem 3.1 imposes.

4. **Conclusions.** This paper has made a useful contribution to the problem of the stability analysis of neutral-type Cohen-Grossberg neural networks possessing multiple time delays in the states of the neurons and multiple neutral delays in time derivative of states of the neurons. By making the use of a suitable Lyapunov functional, this paper has proposed a new sufficient time-independent stability condition for delayed neutral-type Cohen-Grossberg neural networks. The obtained stability criterion can be completely stated in terms of the parameters of the neural network model considered as the proposed criterion only relies on the relationships established among the network parameters. A instructive numerical example has also been given to show the advantages of the derived stability criterion over the previously published stability results for the same class of Cohen-Grossberg neural networks. As pointed out before, obtaining stability conditions for neutral-type Cohen-Grossberg neural networks with multiple delays is a dificult task to achieve.

Therefore, the stability condition given in the current paper makes an important contribution to the stability problem for this class of neutral-type neural networks.

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